

ARO 17126.4-EG

(11)

## REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AD-A117 731		
TITLE (and Subtitle)  FATIGUE OF LAMINATED COMPOSITE STRUCTURES		5. TYPE OF REPORT & PERIOD COVERED  FINAL REPORT 10/24/79 - 4/23/82
		6. PERFORMING ORG. REPORT NUMBER UTEC 82-063
AUTHOR(s)  Dr. George J. Dvorak		7. CONTRACT OR GRANT NUMBER(s)  DAAG29-79-C-0049
PERFORMING ORGANIZATION NAME AND ADDRESS  Department of Civil Engineering University of Utah Salt Lake City, UT 84112		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE  July 1982
MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)  U. S. ARMY RESEARCH OFFICE P. O. BOX 12211 RESEARCH TRIANGLE PARK, NC 27709		13. NUMBER OF PAGES  15
		15. SECURITY CLASS. (of this report)  UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  DTIC ELECTE		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)  S E D		
18. SUPPLEMENTARY NOTES  The view, opinions, and/or findings contained in this report are those of the author and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Composite Materials, Metal Matrices, Fibers, Fatigue		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  SUMMARY  The purpose of the work supported by this contract was to investigate the mechanisms of fatigue in fibrous metal matrix composites and laminates. The approach to the problem was based on two simultaneous research efforts.  In the theoretical part of the program, the elastic-plastic behavior of unidirectional and laminated composites was described with the help of the Vanishing Fiber Diameter (VFD) model. The model simplifies the geometry of		

of the microstructure by assuming that each of the reinforcing fibers has a vanishingly small diameter and that the fibers occupy a finite volume fraction of the composite. Thus the strain fields in both fiber and matrix are uniform. Overall constitutive equations of the elastic-plastic composite aggregate were derived from the properties of the constituents, their volume fractions, and geometry of the microstructure. The theory was extended to the case of in-plane mechanical loading of symmetric laminated plates. Also, a finite element code for three-dimensional analysis of elastic-plastic composite structures was developed and used in selected applications, which included investigations of stress fields at circular holes and cracks in laminated plates. Shakedown limits of certain laminated plates used in the experiments were also established. Work was started on analysis of distributed fracturing in unidirectional composites.

In the experimental part of the program, five 6061-0 Al/B laminated plates of different layup were tested under constant and variable cyclic loading. It was found that when the applied stress range is smaller or equal to the shakedown range of the laminate, there is no detectable damage in the plate specimens after  $2 \times 10^6$  cycles of loading. When the applied stress range exceeds the shakedown range, the aluminum matrix experiences cyclic plastic straining which leads to fatigue crack growth in the matrix, primarily in the off-axis layers, with cracks parallel to the fiber direction. This fatigue cracking causes redistribution of the lamina stresses, and a reduction in the elastic stiffness of the laminate, which can be used as a quantitative measure of fatigue damage. The characteristics, stages and mechanisms of the damage process have been described and analyzed with the plasticity and fracturing theories developed in the theoretical part of the program. The behavior of laminates under variable loading was also studied. It was found that the extent of damage depends primarily on the magnitude of the applied stress range, and in a minor way, on the level of maximum stress.

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UTEC 82-063

FATIGUE OF LAMINATED COMPOSITE STRUCTURES

FINAL REPORT

Period: 24 October 1979-23 April 1982

by

George J. Dvorak

JULY 1982

U. S. ARMY RESEARCH OFFICE

CONTRACT DAAG29-79-C-0049

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## SUMMARY

The purpose of the work supported by this contract was to investigate the mechanisms of fatigue in fibrous metal matrix composites and laminates. The approach to the problem was based on two simultaneous research efforts.

In the theoretical part of the program, the elastic-plastic behavior of unidirectional and laminated composites was described with the help of the Vanishing Fiber Diameter (VFD) model. The model simplifies the geometry of the microstructure by assuming that each of the reinforcing fibers has a vanishingly small diameter and that the fibers occupy a finite volume fraction of the composite. Thus the strain fields in both fiber and matrix are uniform. Overall constitutive equations of the elastic-plastic composite aggregate were derived from the properties of the constituents, their volume fractions, and geometry of the microstructure. The theory was extended to the case of in-plane mechanical loading of symmetric laminated plates. Also, a finite element code for three-dimensional analysis of elastic-plastic composite structures was developed and used in selected applications, which included investigations of stress fields at circular holes and cracks in laminated plates. Shakedown limits of certain laminated plates used in the experiments were also established. Work was started on analysis of distributed fracturing in unidirectional composites.

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causes redistribution of the lamina stresses, and a reduction in the elastic stiffness of the laminate, which can be used as a quantitative measure of fatigue damage. The characteristics, stages and mechanisms of the damage process have been described and analyzed with the plasticity and fracturing theories developed in the theoretical part of the program. The behavior of laminates under variable loading was also studied. It was found that the extent of damage depends primarily on the magnitude of the applied stress range, and in a minor way, on the level of maximum stress.

## 1. ELASTIC-PLASTIC BEHAVIOR OF FIBROUS COMPOSITES AND LAMINATES

In this part of the research work, which is described in References [1] to [7], a comprehensive investigation was made of elastic-plastic deformation of fibrous composites. The fibrous medium was represented by a particular material model, so called Vanishing Fiber Diameter (VFD) model, which assumes that each of the aligned fibers has a vanishingly small diameter, and that the fibers occupy a finite volume fraction of the composite. In this way, the model retains the essential axial constraint between the phases. The local stress and strain fields in the fiber and matrix are uniform, and can be evaluated at each loading step. The macroscopic, or overall response of the composite can be derived from the deformation of the fiber and matrix phases. Overall constitutive equations for a unidirectional fibrous medium have been formulated for the case of transversely isotropic elastic fibers embedded in a Mises-type elastic-plastic matrix. The matrix may exhibit phase hardening, which is either kinematic or isotropic.

The model is especially suitable for analysis of unidirectional and laminated metal matrix composite plates subjected to in-plane mechanical loads. An extension of the theory was made to the case of uniform in-plane loading of symmetric laminates. Constitutive equations for incremental elastic-plastic deformation of the laminated plate were formulated in closed form, and a numerical procedure has been developed for their solution. However, in many problems of practical interest it is necessary to construct plasticity solutions with the help of the finite element method. To make this possible, the constitutive equations for the unidirectional material have been encoded in a finite element program PAC 78. Certain illustrative problems have been solved with this program; those were related to stress concentrations at holes and cracks in laminated plates. Also, the theory was used to establish shakedown envelopes for laminated plates, and the results were applied in analysis of fatigue experiments, as described in the next section.

Thermoplastic behavior of fibrous composites subjected to uniform thermal changes was studied as well. An exact reduction of the thermal problem to a mechanical loading problem was derived for unidirectional fibrous composites of arbitrary geometry in the transverse plane [6]. The phases can be transversely isotropic in their elastic deformation ranges, the fiber remains elastic, the matrix is an elastic-plastic solid with a purely elastic response to hydrostatic stress increments. The reduction permits calculation of instantaneous local stress increments, and overall strain increments, caused by a uniform thermal change, in terms of the instantaneous phase stress concentration factors, and instantaneous compliances, respectively.

The reduction has been applied to the VFD model, and an extension of the theory was made to the case of temperature dependent yield stress. However, the VFD model, in its original form, is not entirely suitable for analysis of thermal problems. The reason is that the loading caused by a uniform thermal change is analogous to macroscopic mechanical loading with a high hydrostatic component. The response of the VFD model to this type of loading is less satisfactory than it is to predominantly plane-stress loading, as explained in [4]. To remove this difficulty, work is underway on more complex composite models which are more appropriate for thermal loading problems.

The plasticity analysis of fibrous metal matrix composites and laminates provides certain generally valid insights into mechanical behavior of these materials. First, the fiber reinforcement increases very significantly both the stiffness and strength of the composite, in the elastic and plastic deformation ranges. However, its influence on the onset of yielding in the matrix is comparatively small. Therefore, plastic deformation is present in a major part of the strength range of the composite. Typically, in 6061-0 Al/B laminates, the initial yield stress is equal to 0.2-0.25 of ultimate strength. This particular aspect of composite behavior makes plasticity analysis indispensable in applications.

The second conclusion that can be derived from the plasticity analysis is that the typical failure strain of composite plates is of the same order of magnitude as the failure strain of the fiber. The fibers used in actual systems can sustain only small strains, usually smaller than 0.01. Under these circumstances, plasticity analysis of metal matrix composite materials must emphasize accuracy in the small strain region. This requires that the theory be based on a micromechanical model which allows for derivation of the overall response from the properties of the constituents and from their mutual constraints which are indicated by the geometry of the microstructure. It is probably obvious that the theory cannot admit certain assumptions which are often accepted in plasticity of metals, or in large strain theories, such as rigid plastic behavior of the phases, inextensibility of the fiber, or plastic incompressibility of the composite medium.

Another aspect of composite and laminate elastic-plastic deformation is the existence of significant hardening which is caused by the interaction between the elastic fiber and the plastically deforming matrix, and is often referred to as constraint hardening. This type of hardening is primarily kinematic, hence the overall yield surface of the composite translates during loading, especially when a normal stress is applied in the axial direction, but it does not expand appreciably. The same is true for shakedown envelopes. This feature of composite behavior is significant under cyclic loading conditions, as described in the next section. Specifically, when the composite is subjected to a cyclic loading program which does not cause shakedown, then extensive fatigue cracking may develop in the matrix and lead to a substantial reduction of elastic composite stiffness. The elastic-plastic analysis makes it clear that the shakedown range will translate, but not expand in the course of loading.

Also, the plasticity analysis of metal matrix laminated plates with stress concentrations caused by holes or cracks [1-3] suggests that plastic yielding does not lead to reduction of local stresses. In fact, as the matrix yields plastically, the fiber stress increases more rapidly with applied load. Under such circumstances, evaluation of fiber stresses with elastic analysis may significantly underestimate the actual values and endanger the safety of the structure.

Finally, it is easy to show that uniform thermal loading of fibrous composites causes large internal stresses, especially in systems with very dissimilar thermal expansion coefficients, e.g., in Gr-Al, or Gr-Mg, but also in B-Al or FP-Al. Accordingly, plastic flow in these materials starts after a moderate thermal change, which is usually smaller than 100°F in systems with annealed aluminum matrices.

All these conclusions indicate that the elastic-plastic behavior of fibrous materials is very different from that of unreinforced metals, and that careful plasticity analysis is required in many applications.

## 2. FATIGUE OF 6061-0 Al/B LAMINATES

Results of fatigue experiments conducted under the contract are described in references [8] to [11]. The fatigue mechanism observed in annealed 6061 Al-B laminates of several different layups can be summarized as follows [6]:

When metal matrix composites are subjected to cyclic loading in the plastic range, they may exhibit two entirely different types of response. If the applied stress amplitude is such that the composite shakes down, then the matrix is strained elastically during the major part of the loading program. In annealed and in some as-fabricated aluminum matrices the matrix endurance limit coincides with the matrix yield stress. Therefore, if the shakedown state is reached, the matrix is stressed within its endurance range and suffers no fatigue damage. The composite then survives  $10^6$ - $10^7$  cycles of loading without damage, providing that the maximum applied stress is such that no significant fiber failure takes place. On the other hand, if the applied stress amplitude is such that the composite does not shake down, then the matrix experiences cyclic plastic straining that leads to low-cycle fatigue damage in the matrix. In the B-Al system with a soft matrix, the fatigue damage is caused by growth of long cracks in the matrix, both in the off-axis and zero-degree plies. The cracks in off-axis plies grow in direction parallel to those of the fibers. In zero degree plies the crack planes are generally perpendicular to the fiber direction, however, the cracks in a soft matrix do not break the large-diameter ( $140 \mu\text{m}$ ), high strength (400 ksi) boron fibers. Instead, the cracks follow a multiply connected path in the matrix between the fibers.

As long as the fibers remain essentially intact, the absence of shakedown and the consequent damage in the matrix may not cause failure of the composite laminate. In fact, fatigue cracking in the off-axis layers reduces stiffness of these layers, and thus causes stress redistribution in the laminate. The weaker laminae are partially unloaded and the load is transferred to the stiffer plies, and particularly to the zero-degree

layers. In many instances this process leads to development of a saturation damage state in which the damaged layers are unloaded to the point where no further fatigue cracking takes place. The damage process is thus arrested and the composite may survive  $10^6$ - $10^7$  cycles of loading after the initial damage period which causes a certain permanent loss of overall stiffness. However, when the stresses are too large for the saturation state to develop, then the damage process continues to the point where the zero-degree plies are overloaded in the course of the internal stress redistribution and the laminate fails. It is expected that a similar fatigue mechanism will be present in other metal matrix composite systems with soft matrices and large-diameter fibers.

The shakedown limits of the laminated plates used in the fatigue tests were evaluated with the elastic-plastic analysis methods discussed in Section 1.

In the test program, unnotched tension coupons 6 in. long and 0.5 in. wide were tested in cyclic tension. Unloading elastic moduli were measured at the onset of cyclic loading and at regular intervals during the program which lasted usually for  $2 \times 10^6$  cycles. It was found that permanent loss of elastic stiffness took place in specimens tested at stress amplitudes exceeding specific magnitudes which were designated as the damage range  $\Delta S_{dm}$ . Comparison of shakedown range predictions with damage range measurements is made in Table 1. In the top part of the table all laminates had zero-degree layers on the surface. The agreement between  $\Delta S_{sh}$  and  $\Delta S_{dm}$  is good. In the bottom part of Table 1, we list results for the  $(90/0)_s$  plate with 90 degree surface layers. The agreement is poor in this case. That may be attributed to the presence of many surface grooves, parallel to fibers, left on the plates after manufacturing. These grooves might have promoted early crack growth in the matrix.

TABLE 1. PREDICTED AND MEASURED B-Al COMPOSITE BEHAVIOR

LAYUP	$c_f$	$\Delta S_{sh}$ Shakedown Range MPa	$\Delta S_{dm}$ Damage Range MPa	$\frac{\Delta S_{dm} - \Delta S_{sh}}{S_{sh}}$
(0) <sub>8</sub>	0.45	429	481	-0.12
(0/90) <sub>2s</sub>	0.50	220	214	+0.03
(0/+45/90) <sub>s</sub>	0.33	183	166	+0.09
(0/+45/90/0/+45/+90) <sub>s</sub>	0.45	191	196	-0.03
(90/0) <sub>2s</sub>	0.50	220	173	+0.21

Figure 1 illustrates many features of fatigue behavior mentioned above. The shakedown region where no fatigue damage takes place occupies about one half of the endurance stress level at  $2 \times 10^6$  cycles. Above the shakedown stress level there is a damage accumulation region, where fatigue cracks start to grow after a certain number of cycles, which depends upon the applied stress magnitude. The dashed lines indicate constant ratios  $E_N/E_0$  of current to initial unloading elastic modulus of the specimen. For example at  $S/S_{ult} = 50\%$ , damage is detected at  $10^4$  cycles, and the modulus drops below 70% of the initial value at  $1 \times 10^6$  cycles. It is obviously desirable to elevate the shakedown stress level towards the S-N curve. This can be achieved by increasing the proportion of zero-degree layers in the laminate.

In addition to the fatigue tests conducted within constant load limits, several tests were performed in which the maximum stress and/or the load range were changed after a certain number of cycles [10, 11]. These results indicate that past cyclic loading history affects the final response, especially the reduction in elastic modulus, in about

the same way as the most severe segment of the total history would if applied above for a sufficiently large number of cycles, typically for about  $0.5 \times 10^6$  cycles. When the applied stress range is kept constant and the maximum stress is gradually increased, each increase leads to a small, abrupt drop in elastic modulus. However, when the maximum stress is kept constant and the stress range is increased, then there is a rapid, large drop in the elastic modulus of the laminate. These results suggest that the magnitude of the stress range has significant influence on the extent of fatigue damage, while the maximum stress level exhibits a minor influence.

Additional work which is still under way includes measurement of crack densities on the surface of unidirectional specimens. These results show that the crack density increases in proportion to applied stress range, and is also proportional to the measured loss in elastic modulus. To establish a predictive relationship between cyclic loading conditions, stiffness reduction and crack density, the fracturing in composite laminates has been analyzed theoretically. Self-consistent estimates of elastic stiffnesses of cracked unidirectional layers were calculated together with average stresses and strains in the fiber and matrix phases of the laminate. These results open the way for introduction of appropriate local fatigue failure criteria, and to extension of the analysis to laminated plates.

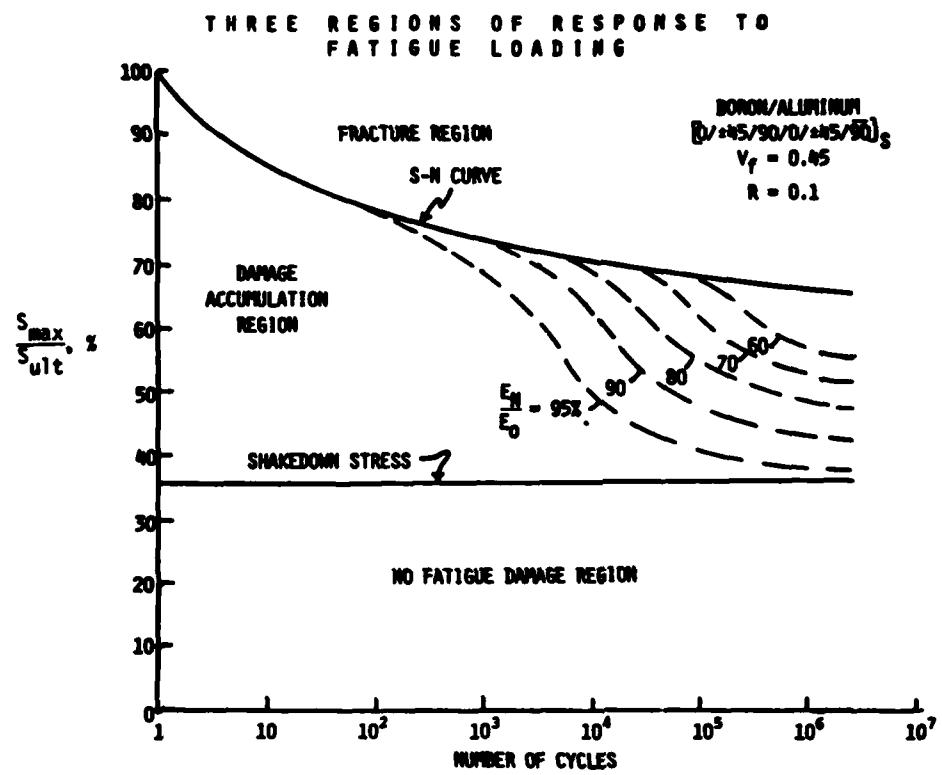


Figure 1. Response of a laminated B-Al plate to cyclic loading.  
 $S_{ult} = 581 \text{ MPa} = 84.26 \text{ ksi}$ .  $E_N/E_0$  is the ratio of current to initial axial elastic modulus of the specimen.

## LIST OF SCIENTIFIC PERSONNEL AND DEGREES EARNED

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### Degrees Granted:

Y. A. Bahei-El-Din	Ph.D., Duke University July 1979 (supported by Grant DAAG29-78-G-0164)
W. S. Johnson	Ph.D., Duke University December 1979

LIST OF PAPERS AND DISSERTATIONS PREPARED

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